Equal Temperment Tuning - Exact Frequencies

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Given any note of a particular frequency, the note exactly one octave above that note is guaranteed to have a frequency exactly twice that of the given note. Mathematically, moving up in pitch requires the frequency to change by multiplying by a scaling factor. Since the note one octave above a given note has twice the original's frequency, then moving up a series of twelve steps individually in order to reach the next octave must return the same frequency that multiplying by 2 would. Therefore, given a starting frequency and the number of tones in an octave, the scaling factor will be whatever number that when multiplied by itself once for every tone in an octave yields the number 2. This is the logic that makes modern Western tuning possible, which has the primary advantage of ensuring that every interval between two notes has the same basic harmonic qualities regardless of where that interval is transposed to. This tuning is very helpful for transposing uniformly, which is part of why it has become the standard tuning. This tuning system, known as Equal Temperment, was an innovative idea at the time of it's conception. Prior to that time, tuning was more inconsistent.

The above logic is basically sufficient to derive the entire formula for finding exact frequencies in any equal temperment system.

The formula for finding the exact frequency of a note in equal temperment tuning (the standard Western tuning) is

$$f_{s,n} = b \cdot (2)^{s/n}$$

where $f_{s,n}$ is the frequency of the note you're trying to find, b is the frequency of the note you are starting on (the base or reference note), n is the number of tones within an octave, and s is the number of half-steps away the note you're trying to find is. The variable s can be positive or negative. Negative values of s mean you're moving down to lower pitches, wheras postive values of s mean you're moving up to higher pitches.

The standard Western tradition is to have 12 tones in every octave. Also, the agreed upon standard for determining the pitches of Western instruments is that A4 (the first A after middle C) is tuned to exactly 440 Hz. Therefore, by combining the above formula with the fact that A4 is 440 Hz, all the frequencies of standard 12-tone equal temperment tuning can be calculated.

In other words, in Western tradition, the equal temperment tuning formula has b set to 440 and n set to 12, giving the more specific formula:

$$f_{s,12} = 440 \cdot (2)^{s/12}$$

These choices of parameters in the Western tradition are very arbitrary, and you can modify the above formulas in order to find the necessary frequencies of any equal temperment tuning system you wish to create.

The standard Western equal temperment tuning system is the most common tuning system, and it's the one you've probably been using and hearing all your life. Here's a complete table of precise values of the frequencies of notes in standard Western Equal Temperment.

Piano Key Number	MIDI Number	Note Name	Sound Frequency (Hz)
1	21	A0	27.5
2	22	A#0	29.13523509488062
3	23	B0	30.86770632850775
4	24	C1	32.70319566257483
5	25	C#1	34.64782887210902
6	26	D1	36.70809598967594
7	27	D#1	38.89087296526011
8	28	E1	41.20344461410874

Piano Key Number	MIDI Number	Note Name	Sound Frequency (Hz)
9	29	F1	43.653528929125486
10	30	F#1	46.24930283895431
11	31	G1	48.999429497718666
12	32	G#1	51.91308719749314
13	33	A1	55
14	34	A#1	58.27047018976124
15	35	B1	61.7354126570155
16	36	C2	65.40639132514966
17	37	C#2	69.29565774421803
18	38	D2	73.41619197935188
19	39	D#2	77.78174593052022
20	40	E2	82.40688922821748
21	41	F2	87.30705785825097
22	42	F#2	92.49860567790861
23	43	G2	97.99885899543733
24	44	G#2	103.82617439498628
25	45	A2	110
26	46	A#2	116.54094037952248
27	47	B2	123.47082531403103
28	48	C3	130.8127826502993
29	49	C#3	138.59131548843604
30	50	D3	146.8323839587038
31	51	D#3	155.56349186104043
32	52	E3	164.81377845643496
33	53	F3	174.61411571650194
34	54	F#3	184.99721135581723
35	55	G3	195.99771799087463
36	56	G#3	207.65234878997256
37	57	A3	220
38	58	A#3	233.08188075904496
39	59	B3	246.94165062806206
40	60	C4 (middle C)	261.6255653005986
41	61	C#4	277.1826309768721
42	62	D4	293.6647679174076
43	63	D#4	311.12698372208087
44	64	E4	329.6275569128699
45	65	F4	349.2282314330039
46	66	F#4	369.99442271163446
47	67	G4	391.99543598174927
48	68	G#4	415.3046975799451
49	69	A4	440
50	70	A#4	466.1637615180899
51	71	B4	493.8833012561241
52	72	C5	523.2511306011972
53	73	C#5	554.3652619537442
54	74	D5	587.3295358348151
55	75	D#5	622.2539674441618
56	76	E5	659.2551138257398
57	77	F5	698.4564628660078
58	78	F#5	739.9888454232689
59	79	G5	783.9908719634986
60	80	G#5	830.6093951598903
61	81	A5	880
62	82	A#5	932.3275230361799

Piano Key Number	MIDI Number	Note Name	Sound Frequency (Hz)
63	83	B5	987.7666025122483
64	84	C6	1046.5022612023945
65	85	C#6	1108.7305239074883
66	86	D6	1174.6590716696303
67	87	D#6	1244.5079348883235
68	88	E6	1318.5102276514797
69	89	F6	1396.9129257320155
70	90	F#6	1479.9776908465378
71	91	G6	1567.981743926997
72	92	G#6	1661.2187903197805
73	93	A6	1760
74	94	A#6	1864.6550460723597
75	95	B6	1975.533205024496
76	96	C7	2093.004522404789
77	97	C#7	2217.461047814977
78	98	D7	2349.31814333926
79	99	D#7	2489.015869776647
80	100	E7	2637.0204553029594
81	101	F7	2793.825851464031
82	102	F#7	2959.9553816930757
83	103	G7	3135.9634878539946
84	104	G#7	3322.437580639561
85	105	A7	3520
86	106	A#7	3729.3100921447194
87	107	B7	3951.066410048992
88	108	C8	4186.009044809578